

MEI STRUCTURED MATHEMATICS

DECISION MATHEMATICS 1, D1

Practice Paper D1-A

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (24 marks)

1 A simple graph is one in which there are no loops, and in which there is no more than one edge connecting any pair of vertices. A particular simple connected graph has 5 vertices and 7 edges, and the order of each vertex is either 2, 3 or 4.

- (i) Explain why the sum of the orders of the vertices is 14. [2]
- (ii) Copy and complete table 1 to show the possibilities for the numbers of vertices of each order.

Number of vertices	Number of order 2	Number of order 3	Number of order 4	Sum of orders
5				14
5				14
5				14

Table 1

- (iii) Draw a diagram for each of your possibilities from part (ii) [3]

2

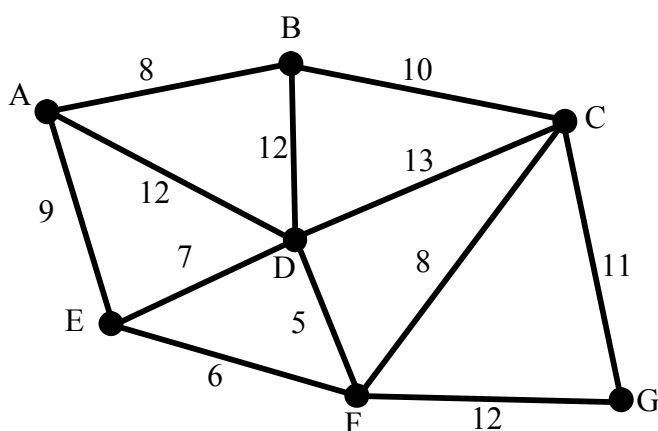


Fig.2

- (a) Starting with vertex A, use Prim's algorithm to find a minimum connector for the network shown in Fig.2.
- (i) Give the order in which you include vertices. [2]
- (ii) Draw your minimum connector and give its total weight. [3]
- (b) (i) Give the order in which arcs are included when Kruskal's algorithm is used to find a minimum connector. [2]
- (ii) Explain why arc DE is not selected when applying Kruskal's algorithm. [1]

- 3 (a) Six items with weights given in the table are to be packed into boxes each of which has a capacity of 10kg.

Item	A	B	C	D	E	F
Weight (kg)	2	1	6	3	3	5

- (i) Use the first-fit algorithm to pack the items, saying how many boxes are needed. [2]
- (ii) Give an optimal solution. [2]
- (b) The following steps define an algorithm which operates on two numbers.
- STEP 1 Write down the two numbers side by side.
- STEP 2 Beneath the left-hand number write down double that number. Beneath the right-hand number write down half of that number, ignoring any remainder.
- STEP 3 Repeat STEP 2 until the right-hand number is 1.
- STEP 4 Delete those rows where the number in the right-hand column is even. Add up the remaining numbers in the left-hand column. This is the result.
- (i) Apply the algorithm to the numbers 50 and 56. [3]
- (ii) Use your result from part (i), and any other simpler examples you may choose, to find what the algorithm achieves. [1]

Section B (48 marks)

- 4 (a) The table shows the activities involved in building a short length of road to bypass a village. The table gives their durations and their immediate predecessors.

Activity	Duration (weeks)	Immediate Predecessors
A Survey sites	8	–
B Purchase land	22	A
C Supply materials	10	–
D Supply machinery	4	–
E Excavate cuttings	9	B, D
F Build bridges and embankments	11	B, C, D
G Lay drains	9	E, F
H Lay hardcore	5	G
I Lay bitumen	3	H
J Install road furniture	10	E, F

- (i) Draw an activity on arc network for these activities. [4]
- (ii) Mark on your diagram the early and late times for each event. Give the minimum completion time and the critical activities. [6]
- (b) Each of the tasks E, F, G and J can be speeded up at extra cost. The maximum number of weeks by which each task can be shortened, and the extra cost per week saved, are shown in the table below.

Task	E	F	G	J
Maximum number of weeks by which task may be shortened	3	3	1	3
Cost per week of shortening task (in thousands of pounds)	30	15	6	20

- (i) Find the new shortest time for the bypass to be completed. [2]
- (ii) List the activities which will need to be speeded up to achieve the shortest time found in part (i), and the times by which each must be shortened. [2]
- (iii) Find the total extra cost needed to achieve the new shortest time. [1]

- 5** The Chief Executive of Leschester City Football Club plc has up to £4 million to spend following a good cup run. He has to decide on spending priorities. Money needs to be spent on strengthening the playing squad and on extra support facilities (i.e. non-playing staff and stadium facilities).

The Coach, who is popular with the fans, has said that he will resign unless he gets at least £2 million to spend on new players.

The authorities require that at least £0.6 million be spent to remedy stadium deficiencies affecting crowd safety.

Club policy is that the amount to be spent on support facilities must be at least one quarter of the amount to be spent on the playing squad.

- (i)** Let $\pounds x$ million be the amount to be spent on the playing squad and let $\pounds y$ million be the amount to be spent on support facilities. Write down four inequalities in terms of x and y representing constraints on spending. [5]
- (ii)** Draw a graph to illustrate your inequalities. [4]
- (iii)** Find the maximum amount which may be spent on the playing squad. [2]

A report commissioned from a market research company indicates that fans regard both team performance and facilities as being important. The report states that the function $0.8x + 0.2y$ gives a measure of satisfaction with extra expenditure.

The Chief Executive proposes to spend £2.5 million on the playing squad and £1.5 million on support facilities

- (iv)** Calculate the measure of satisfaction corresponding to the Chief Executive's proposals. [2]
- (v)** Add to your graph the line $0.8x + 0.2y = 2.3$, and explain what points on this line represent. [2]
- (vi)** The Coach argues that the Chief Executive can achieve the same satisfaction score by spending less in total, but more on the playing squad. How much less and how much more? [2]

- 6 A small ferry boat has spaces for 12 passengers. It arrives at an isolated beach at 3 pm every afternoon, and sets off back to the neighbouring holiday resort at 4 pm. For each of the four 15 minute periods that it waits, the number of passengers arriving is a realisation of a random variable. These random variables are X_1 , X_2 , X_3 and X_4 respectively. Their distributions are given in the tables.

x	0	1	2	3	4
$P(X_1=x)$	0.2	0.2	0.3	0.2	0.1

x	0	1	2	3	4
$P(X_2=x)$	0.1	0.2	0.2	0.3	0.2

x	0	1	2	3	4	5
$P(X_3=x)$	0	0.1	0.1	0.2	0.3	0.3

x	0	1	2	3	4	5	6
$P(X_4=x)$	0	0	0	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

- (i) Give an efficient rule to use 2-digit random numbers to simulate realisations of X_1 . Use the following random numbers to simulate 5 realisations of X_1 .
62 74 91 03 34 [2]
- (ii) Give an efficient rule to use 2-digit random numbers to simulate realisations of X_2 . Use the following random numbers to simulate 5 realisations of X_2 .
60 85 22 50 76 [2]
- (iii) Give an efficient rule to use 2-digit random numbers to simulate realisations of X_3 . Use the following random numbers to simulate 5 realisations of X_3 .
69 01 36 03 93 [2]
- (iv) Give an efficient rule to use 2-digit random numbers to simulate realisations of X_4 . Use the following random numbers to simulate 5 realisations of X_4 .
33 01 99 75 73 89 [3]
- (v) Use your simulated values to simulate the number of passengers the boat carries back on each of 5 afternoons, and hence estimate the mean number carried back. [4]
- (vi) Use your simulated values to estimate the mean number of people per afternoon who fail to secure a place in the boat. [2]
- (vii) How might you improve your estimates in parts (v) and (vi)? [1]